Exercise 33

Evaluate the integral.

$$\int_0^1 (1+r)^3\,dr$$

Solution

Expand the cubic in the integrand; Pascal's triangle gives the coefficients.

1

1 1

1 2 1

1 3 3 1

$$(1+r)^3 = 1\cdot 1^3 r^0 + 3\cdot 1^2 r^1 + 3\cdot 1^1 r^2 + 1\cdot 1^0 r^3 = 1 + 3r + 3r^2 + r^3$$

Use the properties of integrals to write the given integral as integrals of monomials. Then integrate by using the power rule in reverse: Bump up the exponent by 1 and divide by that same number.

$$\begin{aligned}
\int_0^1 (1+r)^3\,dr &= \int_0^1 (1+3r+3r^2+r^3)\,dr \\
&= \int_0^1 1\,dr + \int_0^1 3r\,dr + \int_0^1 3r^2\,dr + \int_0^1 r^3\,dr \\
&= \int_0^1 1\,dr + 3\int_0^1 r\,dr + 3\int_0^1 r^2\,dr + \int_0^1 r^3\,dr \\
&= (r)\bigg|_0^1 + 3\left(\frac{r^2}{2}\right)\bigg|_0^1 + 3\left(\frac{r^3}{3}\right)\bigg|_0^1 + \left(\frac{r^4}{4}\right)\bigg|_0^1 \\
&= (1-0) + 3\left(\frac{1^2}{2}-\frac{0^2}{2}\right) + 3\left(\frac{1^3}{3}-\frac{0^3}{3}\right) + \left(\frac{1^4}{4}-\frac{0^4}{4}\right) \\
&= 1 + \frac{3}{2} + 1 + \frac{1}{4} \\
&= \frac{15}{4}
\end{aligned}$$